The Use of Morlet Wavelet Coefficients for Identifying Fatigue Damage Features

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ABSTRACT
This paper presents a new approach to identify fatigue damaging potential locations using the Morlet wavelet coefficients. For solving the subject matter, the 122.4 second SAESUS strain signal was selected for the simulation purpose. As the result, the Morlet wavelet coefficients predicted that the maximum fatigue damage occurs at 40.4 - 42.6 seconds and 67.4 - 70 seconds. For the validation purpose, the Morrow’s fatigue damaging value was calculated and was obtained that the maximum fatigue damage occurs at 0 seconds and 99.7 seconds. The fatigue damaging value at the points was 0.0047 cycles to failure. Since both the plots had similar pattern, the Morlet wavelet coefficients could be used as the early warning of the fatigue damaging potential locations, although the locations were not entirely correct.

Keywords: Fatigue strain signal, Morlet wavelet coefficient, fatigue damage

INTRODUCTION
August Wöhler was the first engineer to study fatigue failure and proposed an empirical analysis technique. Between 1852 and 1870, he studied the progressive failure of railway axles. Two railways were suspended from the ends of the axles and the axles rotated till failure. He then plotted the nominal stress versus the numbers of rotation to failure on what has become known as the stress-life (S-N) diagram. The S-N - analysis is valid between the transition and the endurance limit (approximately 10⁶ cycles for steel). Above the endurance limit, the slope of the curve reduces dramatically and as such this is often referred to as the ‘infinite life’ region. Several effects are notable about the approach. At below the transition point (approximately 1000 cycles), the S-N curve is not valid because the nominal stresses are now elastic-plastic. For this case, the strain-life (-N) - based approach is appropriate method and is commonly used to predict fatigue life for ductile materials at relatively short fatigue life. The crack initiation method relates the plastic deformation that occurs at a localized region where fatigue cracks begin to the durability of the structure under influence of mean stress.

Most fatigue life predictions are based on the Palmgren-Miner’s linear cumulative damaging rule normally applied with the established strain-life fatigue damaging models. However, several limitations were found in the implementation of the rule. Using this approach, the fatigue damage is accurately calculated for constant amplitude loadings, but it may lead to the erroneous prediction for variable amplitude loading (Fatemi and Yang, 1998). Such rule assumes no load sequence effect and does not consider the load-interaction accountability that occurs in fatigue service loadings.
Considering the importance of the influence of load sequence effects and the limitation of the rule, therefore, a suitable improved approach of fatigue live prediction for analyzing components subjected to variable amplitude loadings needs to be identified.

This paper presents a novel approach to indentify the fatigue damaging potential locations. This method utilized the Morlet wavelet coefficients which indicate how energy in the signal is distributed in the time-frequency plane (Darpe, 2007). The energy spectrum (the energy density over frequency) is plotted in order to observe the signal behaviour and its content gives significant information about the random signal pattern. This study uses the Morlet wavelet coefficients to indentify the fatigue damage features based on the wavelet coefficient plot. The schematic flow of the process is illustrated in Fig. 1. The strain signal selected for the simulation purpose was obtained from the database of Society of Automotive Engineers (SAE) profiles, named the SAESUS. The signal (in the unit of microstrain) was collected from a suspension component of a car and it was assumed to be sampled at 204.8 Hz for 25,061 data points. It gave the total record length of the signal of 122.4 seconds.

**ANALYSIS OF THE MORLET WAVELET COEFFICIENT**

The Wavelet Transform (WT) approach is probably the most recent solution to overcome the nonstationary signals. This time-frequency technique is applied by cutting time domain signal into various frequency components through the compromise between time and frequency - based views of the signal. It presents information in both time and frequency domain in a more useful form (Valens, 1999; Addison, 2002; Percival and Walden, 2000).

The analysis is started with a basic function (called the mother wavelet) scaled and translated to represent the signal being analyzed (Berry, 1999). The transform shifts a window along the signal and calculates the spectrum for every position. The process is repeated many times with a slightly shorter (or longer) window for every new cycle. In the end, the result will be a collection of time-frequency representations of the signal with different resolutions. The WT provides information on when and at what frequency the signal occurs (Valens, 1999). Obviously, the WT represents a windowing technique with variable-sized regions. This technique allows the use of long time intervals (more precise low frequency information) and shorter regions (high frequency...
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information). The major advantage is the ability to analyze a localized area of larger signal (local analysis) (Misiti et al., 2008).

The Continuous Wavelet Transform (CWT) can operate at every possible scale, generates an awful lot of data, and is used to know all values of a continuous decomposition to reconstruct the signal exactly. In addition, it is easier to interpret, makes all information more visible, and is sufficient for exact reconstruction. The Morlet wavelet is one of functions which are the most generally used in the CWT analyses (Gao et al., 2001). The name of the wavelet family is written “morl”. The wavelet decomposition calculates a resemblance index between signal being analyzed and the wavelet, called coefficient. It is a result of a regression of an original signal produced at different scales and different sections on the wavelet. It represents correlation between the wavelet and a section of the signal. If the index is large, the resemblance is strong, otherwise it is slight (Misiti et al., 2008).

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The WT of any time-varying signal \( f(t) \) is defined as the sum of all of the signal time multiplied by a scaled and shifted version of the wavelet function \( \psi(t) \) (Kim et al., 2007). The CWT is expressed by the following integral:

\[
\text{CWT}_{(a,b)} = \int_{-\infty}^{+\infty} f(t) \psi_{a,b}(t) dt
\]

The parameter \( a \) represents the scale factor which is a reciprocal of frequency, the parameter \( b \) indicates the time shifting or translation factor, and \( t \) is time.

\[
\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right) a, b \in R; a \neq 0
\]

The original signal was transformed into the Morlet wavelet using the CWT. This algorithm presented the distribution of the wavelet coefficients in time-frequency domain. Fig. 2 shows the SAESUS strain signal and the wavelet coefficient distribution. In the scalogram, the \( x \)-axis denoted the time parameter, the \( y \)-axis represented the scale that has an inversely related to the frequency value, and the colour intensity at each \( x-y \) point was proportional to the absolute value of the wavelet coefficients as a function of the dilation and translation parameters. It provided the energy distribution display with respect to the particular time and frequency information. Accordingly, a lower scale indicated higher frequency and had small amplitude that means that these cycles had lower energy. They gave minimal or no fatigue damaging potential. A large scale was indicative of lower frequency and higher amplitude that indicates that these cycles had higher energy causing the fatigue damage.

Using the newly Morlet wavelet - based developed computational algorithm, the wavelet coefficient magnitude segments were transposed into time domain SAESUS signal, as presented in Fig. 3. The representation showed a two dimensional view of the energy distribution, as observed in time-frequency plane. On this plot, the higher coefficients occur at 40.4 - 42.6 seconds and 67.4 - 70 seconds. Since the higher Morlet wavelet coefficients indicate higher fatigue damage, it indicates that the fatigue damaging events also will occur at the same points.
FATIGUE LIFE PREDICTION

For the validation purpose, the fatigue damaging value was estimated using a specific commercial software package. Comparing the Morlet wavelet coefficients and the fatigue damage, it was obtained the effectiveness of the wavelet coefficients in identifying the fatigue damaging potential locations. The signal is compressed data since the mean value is minus. In a case of the loading being predominantly compressive, particularly for wholly compressive cycles, the Morrow’s model provides more realistic live estimates. The mean stress correction effect seems to work reasonably well for steels. The model is mathematically defined as the following expression (nCode):

\[
\varepsilon_a = \frac{\sigma_f'}{E} \left(1 - \frac{\sigma_m}{\sigma_f'}\right)(2N_r)^b + \varepsilon_f' (2N_r)^c
\]

where \( \varepsilon_a \) is the true strain amplitude, \( \sigma_f' \) is the fatigue strength coefficient, \( E \) is the material modulus of elasticity, \( \sigma_m \) is the mean stress, \( N_r \) is the numbers of cycle to failure for a particular stress range and mean, \( b \) is the fatigue strength exponent, \( \varepsilon_f' \) is the fatigue ductility coefficient, and \( c \) is the fatigue ductility exponent.
The fatigue damage caused by each cycle of repeated loading is calculated by reference to material live curves, such as $S-N$ or $\varepsilon-N$ curves. The fatigue damage $D$ for one cycle and the total fatigue damage $\Sigma D$ caused by cycles are expressed respectively as (Abdullah, 2005):

$$D = \frac{1}{N_i}$$  \hspace{1cm} (5)

$$\Sigma D = \sum \frac{N_i}{N_j}$$  \hspace{1cm} (6)

where $D_{N_i}$ is the numbers of cycle within a particular stress range and mean.

For the fatigue damaging calculation, the selected material for the simulation purpose was the SAE1045 carbon steel shaft. This material was chosen because it was commonly used in automotive industries for fabricate a vehicle lower suspension arm structure (Khalil and Topper, 2003). The material properties and their definitions are given in Table 1 (nCode, 2005).

From the fatigue damaging analysis using the Morrow’s strain-life model, it was obtained that the maximum fatigue damaging events occur at 0 seconds and 99.7 seconds. The fatigue damaging value at the points was 0.0047 cycles to failure. The fatigue damaging distribution is presented in Fig. 4 and Fig. 5 plots the fatigue damaging and cycle counting histograms.

<table>
<thead>
<tr>
<th>Properties</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ultimate tensile strength, $S_u$ (MPa)</td>
<td>621</td>
</tr>
<tr>
<td>Modulus of elasticity, $E$ (GPa)</td>
<td>204</td>
</tr>
<tr>
<td>Fatigue strength coefficient, $\sigma'_f$ (MPa)</td>
<td>948</td>
</tr>
<tr>
<td>Fatigue strength exponent, $b$</td>
<td>-0.092</td>
</tr>
<tr>
<td>Fatigue ductility exponent, $c$</td>
<td>-0.445</td>
</tr>
<tr>
<td>Fatigue ductility coefficient, $\varepsilon'_f$</td>
<td>0.26</td>
</tr>
</tbody>
</table>

**Fig. 3:** The Morlet wavelet coefficients in time representation

**Fig. 4:** Time series of the Morrow’s fatigue damaging events
IDENTIFICATION OF FATIGUE DAMAGING FEATURES

According to the analysis findings, the Morlet wavelet coefficients have been found to be able in identifying the fatigue damaging potential locations. Although the fatigue damaging events predicted by the Morlet wavelet coefficients were not entirely correct, but they can be used as the early warning of the fatigue damaging locations. Fig. 6 shows the plot of the Morlet wavelet coefficients when compared to the fatigue damaging event plot, and both plots had similar pattern. Therefore, the resemblance index representing correlation between the Morlet wavelet and the signal was strong.

CONCLUSION

This paper discussed on the effectiveness of the Morlet wavelet coefficients in predicting the fatigue damaging potential locations. The used signal was obtained from the database of SAE profiles, named the SAESUS. In overall, this study found that the Morlet wavelet coefficients can be used to locate the fatigue damaging events. Although the results were not entirely correct, but it could be used as the early warning of the fatigue damaging potential locations, since the Morlet wavelet coefficient plot type was similar to the fatigue damaging plot pattern.
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In the future, the authors suggest some works related to development of a relationship model between the Morlet wavelet coefficients and the fatigue damage. In this aspect, it was suggested that the wavelet coefficients are not only used to predict the locations, but the wavelet also can be used to predict the fatigue damaging values.

![Fig. 6: Plot comparison: (a) The Morlet wavelet coefficients, and (b) the fatigue damaging events](image)

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